

# A Study of $B_d^0 \rightarrow J/\Psi\eta^{(\prime)}$ Decays in the pQCD Approach

Xin Liu<sup>a\*</sup>, Zhen-Jun Xiao<sup>b†</sup>, Hui-Sheng Wang<sup>c</sup>

*a. Department of Physics, Zhejiang Ocean University,  
Zhoushan, Zhejiang 316000, P.R. China*

*b. Department of Physics and Institute of Theoretical Physics,  
Nanjing Normal University, Nanjing, Jiangsu 210097, P.R. China and*

*c. Department of Applied Mathematics and Physics,  
Anhui University of Technology and Science,  
Wuhu, Anhui 241000, P.R. China*

(Dated: February 1, 2008)

## Abstract

Motivated by the very recent measurement of the branching ratio of  $B_d^0 \rightarrow J/\psi\eta$  decay, we calculate the branching ratios of  $B_d^0 \rightarrow J/\psi\eta$  and  $B_d^0 \rightarrow J/\Psi\eta'$  decays in the perturbative QCD (pQCD) approach. The pQCD predictions for the branching ratios of considered decays are:  $BR(B_d^0 \rightarrow J/\Psi\eta) = (1.96_{-0.65}^{+9.68}) \times 10^{-6}$ , which is consistent with the first experimental measurement within errors; while  $BR(B_d^0 \rightarrow J/\Psi\eta') = (1.09_{-0.25}^{+3.76}) \times 10^{-6}$ , very similar with  $B_d^0 \rightarrow J/\Psi\eta$  decay and can be tested by the forthcoming LHC experiments. The measurements of these decay channels may help us to understand the QCD dynamics in the corresponding energy scale, especially the reliability of pQCD approach to these kinds of B meson decays.

PACS numbers: 13.25.Hw, 12.38.Bx, 14.40.Nd

---

\* liuxin@zjou.edu.cn

† xiaozhenjun@njnu.edu.cn

Very recently, the first observation of  $B_d^0 \rightarrow J/\Psi\eta$  decay was reported by Belle Collaboration [1], and the branching ratio measured is

$$BR(B_d^0 \rightarrow J/\Psi\eta) = (9.5 \pm 1.7(stat) \pm 0.8(syst)) \times 10^{-6}, \quad (1)$$

which is consistent with the currently available theoretical predictions [1, 2, 3].

Up to now, the theoretical calculations for the branching ratios of  $B_d \rightarrow J/\Psi\eta^{(\prime)}$  decays were obtained by using the heavy quark factorization approximation in Ref. [2], or from the measured  $J/\Psi\pi^0$  and  $J/\Psi K^0$  branching ratios[3, 4, 5] based on the assumption of the  $SU(3)$  flavor symmetry of strong interaction. In this paper, we will calculate the branching ratios of  $B_d^0 \rightarrow J/\Psi\eta$  and  $B_d^0 \rightarrow J/\Psi\eta^{(\prime)}$  decays directly by employing the low energy effective Hamiltonian [6] and the perturbative QCD (pQCD) factorization approach [7, 8, 9].

The paper is organized as follows: we present the formalism used in the calculation of  $B_d^0 \rightarrow J/\psi\eta^{(\prime)}$  decays in Sec. I. In Sec. II, we show the numerical results and compare them with the measured values. A short summery and some conclusions are also included in this section.

## I. FORMALISM AND PERTURBATIVE CALCULATIONS

The pQCD approach has been developed earlier from the QCD hard-scattering approach [7], and has been used frequently to calculate various B meson decay channels [7, 8, 9, 10]. For two body charmless hadronic  $B_{d,s} \rightarrow M\eta^{(\prime)}$  (here  $M$  stands for the pseudo-scalar or vector light mesons composed of the light quarks  $u, d, s$ ) decays, the pQCD predictions generally agree well with the measured values [9, 10, 11].

In Refs. [12, 13], the authors calculated  $B \rightarrow D_s^* K, D_s^{(*)+} D_s^{(*)-}$  and  $B_s \rightarrow D^{(*)+} D^{(*)-}$  decays and found that the pQCD approach works well for such decays. Here we try to apply the pQCD approach to calculate the B meson decays involving the heavier  $J/\Psi$  meson as one of the two final state mesons.

### A. Formalism

In pQCD approach, the decay amplitude of  $B \rightarrow J/\Psi P$  ( $P = \eta, \eta^{(\prime)}$  here) decay can be written conceptually as the convolution,

$$\mathcal{A}(B \rightarrow M_1 M_2) \sim \int d^4 k_1 d^4 k_2 d^4 k_3 \text{Tr} [C(t) \Phi_B(k_1) \Phi_{J/\Psi}(k_2) \Phi_P(k_3) H(k_1, k_2, k_3, t)], \quad (2)$$

where the term “Tr” denotes the trace over Dirac and color indices.  $C(t)$  is the Wilson coefficient which results from the radiative corrections at short distance. In the above convolution,  $C(t)$  includes the harder dynamics at larger scale than  $M_B$  scale and describes the evolution of local 4-Fermi operators from  $m_W$  (the  $W$  boson mass) down to  $t \sim \mathcal{O}(\sqrt{\bar{\Lambda} M_B})$  scale, where  $\bar{\Lambda} \equiv M_B - m_b$ . The function  $H(k_1, k_2, k_3, t)$  is the hard part and can be calculated perturbatively. The function  $\Phi_M$  is the wave function which describes hadronization of the quark and anti-quark to the meson  $M$ . While the function  $H$  depends on the process considered, the wave function  $\Phi_M$  is independent of the specific process.

Using the wave functions determined from other well measured processes, one can make quantitative predictions here.

Using the light-cone coordinates the  $B$  meson and the two final state meson momenta can be written as

$$P_1 = \frac{M_B}{\sqrt{2}}(1, 1, \mathbf{0}_T), \quad P_2 = \frac{M_B}{\sqrt{2}}(1, r^2, \mathbf{0}_T), \quad P_3 = \frac{M_B}{\sqrt{2}}(0, 1 - r^2, \mathbf{0}_T), \quad (3)$$

respectively, where  $r = M_{J/\Psi}/M_B$ , and the light meson masses  $m_\eta^{(\prime)}$  have been neglected. The longitudinal polarization vector of the  $J/\Psi$  meson,  $\epsilon_L$ , is given by  $\epsilon_L = \frac{M_B}{\sqrt{2}M_{J/\Psi}}(1, -r^2, \mathbf{0}_T)$ . Putting the light (anti-) quark momenta in  $B$ ,  $J/\Psi$  and  $\eta^{(\prime)}$  mesons as  $k_1$ ,  $k_2$ , and  $k_3$ , respectively, we can choose

$$k_1 = (x_1 P_1^+, 0, \mathbf{k}_{1T}), \quad k_2 = (x_2 P_2^+, 0, \mathbf{k}_{2T}), \quad k_3 = (0, x_3 P_3^-, \mathbf{k}_{3T}). \quad (4)$$

Then, for  $B \rightarrow J/\Psi \eta$  decay for example, the integration over  $k_1^-$ ,  $k_2^-$ , and  $k_3^+$  in eq.(2) will lead to

$$\mathcal{A}(B \rightarrow J/\Psi \eta') \sim \int dx_1 dx_2 dx_3 b_1 db_1 b_2 db_2 b_3 db_3 \cdot \text{Tr} [C(t) \Phi_B(x_1, b_1) \Phi_{J/\Psi}(x_2, b_2) \Phi_\eta(x_3, b_3) H(x_i, b_i, t) S_t(x_i) e^{-S(t)}], \quad (5)$$

where  $b_i$  is the conjugate space coordinate of  $k_{iT}$ , and  $t$  is the largest energy scale in function  $H(x_i, b_i, t)$ . The large logarithms  $\ln(m_W/t)$  are included in the Wilson coefficients  $C(t)$ . The large double logarithms ( $\ln^2 x_i$ ) on the longitudinal direction are summed by the threshold resummation [14], and they lead to  $S_t(x_i)$  which smears the end-point singularities on  $x_i$ . The last term,  $e^{-S(t)}$ , is the Sudakov form factor which suppresses the soft dynamics effectively [15]. Thus it makes the perturbative calculation of the hard part  $H$  applicable at intermediate scale, i.e.,  $M_B$  scale. We will calculate analytically the function  $H(x_i, b_i, t)$  for the considered decays in the first order in  $\alpha_s$  expansion and give the convoluted amplitudes in next section.

## B. The $B_d^0 \rightarrow J/\Psi \eta^{(\prime)}$ Decays

The low energy effective Hamiltonian for decay modes  $B_d^0 \rightarrow J/\psi \eta^{(\prime)}$  can be written as

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} [V_{cb} V_{cd}^* (C_1(\mu) O_1^c(\mu) + C_2(\mu) O_2^c(\mu))] , \quad (6)$$

with the four-fermion operators

$$O_1^c = \bar{d}_\alpha \gamma^\mu (1 - \gamma_5) c_\beta \cdot \bar{c}_\beta \gamma_\mu (1 - \gamma_5) b_\alpha, \quad O_2^c = \bar{d}_\alpha \gamma^\mu (1 - \gamma_5) c_\alpha \cdot \bar{c}_\beta \gamma_\mu (1 - \gamma_5) b_\beta \quad (7)$$

where the Wilson coefficients  $C_i(\mu)$  ( $i = 1, 2$ ), we will use the leading order (LO) expressions, although the next-to-leading order (NLO) results already exist in the literature [6]. This is the consistent way to cancel the explicit  $\mu$  dependence in the theoretical formulae. For the renormalization group evolution of the Wilson coefficients from higher scale to lower scale, we use the formulae as given in Ref.[16] directly.

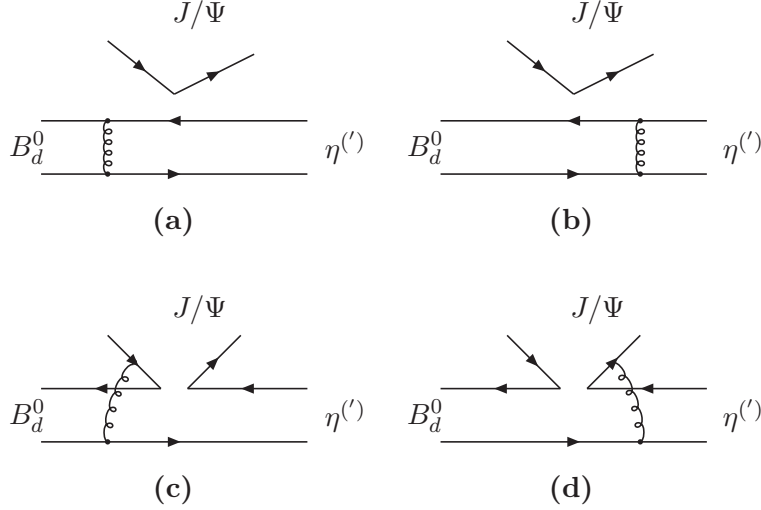


FIG. 1: Typical Feynman diagrams contributing to the Cabibbo- and color- suppressed  $B_d^0 \rightarrow J/\psi \eta'$  decays.

As for  $B$  meson wavefunction, we make use of the same parameterizations as used in the studies of different processes [16]. For vector  $J/\psi$  meson, in terms of the notation in Ref. [17], we decompose the nonlocal matrix elements for the longitudinally and transversely polarized  $J/\psi$  mesons into

$$\Phi_{J/\psi}(x) = \frac{1}{\sqrt{2N_c}} \left\{ m_{J/\psi} \not{\epsilon}_L \Psi^L(x) + \not{\epsilon}_L \not{P} \Psi^t(x) \right\}, \quad (8)$$

Here,  $\Psi^L$  denote for the twist-2 distribution amplitudes, and  $\Psi^t$  for the twist-3 distribution amplitudes.  $x$  represents the momentum fraction of the charm quark inside the charmonium.

The  $J/\psi$  meson asymptotic distribution amplitudes read as [18]

$$\begin{aligned} \Psi^L(x) &= 9.58 \frac{f_{J/\psi}}{2\sqrt{2N_c}} x(1-x) \left[ \frac{x(1-x)}{1-2.8x(1-x)} \right]^{0.7}, \\ \Psi^t(x) &= 10.94 \frac{f_{J/\psi}}{2\sqrt{2N_c}} (1-2x)^2 \left[ \frac{x(1-x)}{1-2.8x(1-x)} \right]^{0.7}. \end{aligned} \quad (9)$$

It is easy to see that both the twist-2 and twist-3 DAs vanish at the end points due to the factor  $[x(1-x)]^{0.7}$ .

From the effective Hamiltonian (6), the Feynman diagrams corresponding to the considered decay are shown in Fig.1. With the meson wave functions and Sudakov factors,

the hard amplitude is given as

$$\begin{aligned}
F_{e\eta} = & 8\pi C_F m_B^4 \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_B(x_1, b_1) \\
& \times \{ (1-r^2) [(1+x_3(1-r^2))\phi_\eta^A(x_3, b_3) + r_0(1-2x_3) \\
& \cdot \phi_\eta^P(x_3, b_3)] + r_0 [(1-2x_3) + r^2(1+2x_3)] \phi_\eta^T(x_3, b_3) \\
& \cdot \alpha_s(t_e^1) h_e(x_1, x_3, b_1, b_3) \exp[-S_{ab}(t_e^1)] \\
& + 2r_0 [1 - (1-x_1)r^2\phi_\eta^P(x_3, b_3) - x_1r^2\phi_\eta^A(x_3, b_3)] \\
& \cdot \alpha_s(t_e^2) h_e(x_3, x_1, b_3, b_1) \exp[-S_{ab}(t_e^2)] \} .
\end{aligned} \tag{10}$$

where  $r_0 = m_0^\eta/m_B$ ;  $C_F = 4/3$  is a color factor. The function  $h_e$ , the scales  $t_e^i$  and the Sudakov factors  $S_{ab}$  are displayed in Appendix A.

For the non-factorizable diagrams 1(c) and 1(d), all three meson wave functions are involved. The integration of  $b_3$  can be performed using  $\delta$  function  $\delta(b_3 - b_1)$ , leaving only integration of  $b_1$  and  $b_2$ . For the concerned operators, the corresponding decay amplitude is

$$\begin{aligned}
M_{e\eta} = & \frac{16\sqrt{6}}{3} \pi C_F m_B^4 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_s}(x_1, b_1) \\
& \times \{ 2rr_c \phi_{J/\Psi}^t(x_2, b_2) \phi_\eta^A(x_3, b_2) - 4rr_0 r_c \phi_{J/\Psi}^t(x_2, b_2) \phi_\eta^T(x_3, b_2) \\
& - [x_2 r^2 + x_3(1-2r^2)] \phi_{J/\Psi}^L(x_2, b_2) \phi_\eta^A(x_3, b_2) \\
& + 2[x_3 r_0 + (x_2 - x_3)r_0 r^2] \phi_{J/\Psi}^L(x_2, b_2) \phi_\eta^T(x_3, b_2) \\
& \cdot \alpha_s(t_f) h_f(x_1, x_2, x_3, b_1, b_2) \exp[-S_{cd}(t_f)] \} .
\end{aligned} \tag{11}$$

where  $r_c = m_c/m_B$ ,  $m_c$  is the mass for  $c$  quark.

For the  $B_d^0 \rightarrow J/\Psi \eta'$  decay, the Feynman diagrams are obtained by replacing the  $\eta$  meson in Fig. 1 with the meson  $\eta'$ . The corresponding expressions of decay amplitudes will be similar with those as given in Eqs.(10-11), since the  $\eta$  and  $\eta'$  are all light pseudoscalar mesons and have the similar wave functions. The expressions of  $B_d^0 \rightarrow J/\Psi \eta'$  decay can be obtained simply by the following replacements

$$\phi_\eta^A \longrightarrow \phi_{\eta'}^A, \quad \phi_\eta^P \longrightarrow \phi_{\eta'}^P, \quad \phi_\eta^T \longrightarrow \phi_{\eta'}^T, \quad r_0 \longrightarrow r'_0. \tag{12}$$

For the  $\eta - \eta'$  system, there exist two popular mixing basis: the octet-singlet basis and the quark-flavor basis [19, 20]. Here we use the quark-flavor basis [19] and define

$$\eta_q = (u\bar{u} + d\bar{d})/\sqrt{2}, \quad \eta_s = s\bar{s}. \tag{13}$$

The physical states  $\eta$  and  $\eta'$  are related to  $\eta_q$  and  $\eta_s$  through a single mixing angle  $\phi$ ,

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = U(\phi) \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}. \tag{14}$$

The three input parameters  $f_q$ ,  $f_s$  and  $\phi$  in the quark-flavor basis have been extracted from various related experiments [19, 20]

$$f_q = (1.07 \pm 0.02) f_\pi, \quad f_s = (1.34 \pm 0.06) f_\pi, \quad \phi = 39.3^\circ \pm 1.0^\circ, \tag{15}$$

where  $f_\pi = 130$  MeV. In the numerical calculations, we will use these mixing parameters as inputs. It is worth mentioning that the effects of possible gluonic component of  $\eta'$  meson will not be considered here since it is small in size [10, 21, 22].

For  $B_d^0 \rightarrow J/\Psi\eta$  decay, by combining the contributions from different diagrams, the total decay amplitude can be written as

$$\mathcal{M}(B_d^0 \rightarrow J/\Psi\eta) = V_{cb}V_{cd}^*F_1(\phi) \left\{ F_{e\eta}f_{J/\Psi} \left[ \left( C_1 + \frac{1}{3}C_2 \right) \right] + M_{e\eta}C_2 \right\} \quad (16)$$

where the relevant mixing parameter is  $F_1(\phi) = \cos \phi/\sqrt{2}$ .

It should be mentioned that the Wilson coefficients  $C_i = C_i(t)$  in Eq. (16) should be calculated at the appropriate scale  $t$  using equations as given in the Appendices of Ref. [16]. Here the scale  $t$  in the Wilson coefficients should be taken as the same scale appeared in the expressions of decay amplitudes in Eqs. (10) and (11). This is the way in pQCD approach to eliminate the scale dependence. In order to estimate the effect of higher order corrections, however, we introduce a scale factor  $a_t = 1.0 \pm 0.2$  and vary the scale  $t_{max}$  as described in Appendix A.

Similarly, the decay amplitudes for  $B_d^0 \rightarrow J/\Psi\eta'$  decay can be obtained easily from Eq.(16) by the following replacements of  $F_1(\phi) \rightarrow F'_1(\phi) = \sin \phi/\sqrt{2}$ .

## II. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we will calculate the branching ratios for those considered decay modes. The input parameters and the wave functions to be used are given in Appendix B. In numerical calculations, central values of input parameters will be used implicitly unless otherwise stated.

With the complete decay amplitudes, we can obtain the decay width for the considered decays,

$$\Gamma(B_d^0 \rightarrow J/\psi\eta^{(\prime)}) = \frac{G_F^2 M_B^3}{32\pi} (1 - r^2) \left| \mathcal{M}(B_d^0 \rightarrow J/\psi\eta^{(\prime)}) \right|^2. \quad (17)$$

By employing the quark-flavor scheme of  $\eta-\eta'$  system and using the mixing parameters as given in Eq. (15), one finds the branching ratios for the considered two decays with error bars as follows:

$$Br(B_d^0 \rightarrow J/\Psi\eta) = [1.96_{-0.50}^{+0.71}(\omega_b)_{-0.39}^{+9.65}(a_t)_{+0.13}^{+0.32}(a_2)_{-0.13}^{+0.14}(f_{J/\Psi})] \times 10^{-6}, \quad (18)$$

$$Br(B_d^0 \rightarrow J/\Psi\eta') = [1.09_{-0.24}^{+0.32}(\omega_b)_{+0.01}^{+3.73}(a_t)_{+0.01}^{+0.28}(a_2)_{-0.07}^{+0.08}(f_{J/\Psi})] \times 10^{-6}, \quad (19)$$

where the main errors are induced by the uncertainties of  $\omega_b = 0.40 \pm 0.05$  GeV,  $a_t = 1.0 \pm 0.2$ ,  $a_2 = 0.115 \pm 0.115$  and  $f_{J/\Psi} = 0.405 \pm 0.014$  GeV, respectively. One can see that the pQCD predictions are sensitive to the variations of  $\omega_b$  and  $a_t$ .

For  $B_d^0 \rightarrow J/\Psi\eta$  decay, the central value of the pQCD prediction for  $Br(B_d^0 \rightarrow J/\Psi\eta)$  is a factor of 4 smaller than the measured value as given in Eq. (1) [1]. But the pQCD prediction is in fact still consistent with Belle's first measurement if we take the large theoretical and experimental errors into account. By varying the scale factor  $a_t$  in the range of  $a_t = [0.8, 1.0]$ , for example, the central value of  $Br(B \rightarrow J/\Psi\eta)$  will change in the

range of  $[0.2, 1.1] \times 10^{-5}$  accordingly. It is not difficult to understand such  $a_t$  dependence. Since the  $J/\Psi$  meson is much heavier than light mesons, and therefore moving not as fast as those light meson when B meson is decaying. So a small decrease of the scale  $t_i$  will lead to a larger Wilson coefficients  $C_{1,2}(t)$  and  $\alpha_s(t_i)$ , and consequently results in a larger decay rate.

For  $B_d^0 \rightarrow J/\Psi\eta'$  decay, only experimental upper limit (at 90% C.L) is available now:  $BR(B^0 \rightarrow J/\Psi\eta') < 6.3 \times 10^{-5}$  [4, 5]. The pQCD prediction for the branching ratio of  $B_d^0 \rightarrow J/\Psi\eta'$  decay is very similar in magnitude with that of  $B_d^0 \rightarrow J/\Psi\eta$ , consistent with the upper limit and will be tested in the forthcoming LHC experiments.

At the leading order, only the tree Feynman diagrams as shown in Fig. 1 contribute to  $B_d^0 \rightarrow J/\Psi\eta^{(\prime)}$  decays. There exists no CP violation in these decays within the standard model, since there is only one kind of Cabibbo-Kabayashi-Muskawa (CKM) phase involved in the corresponding decay amplitudes, as can be seen from eq. (16).

In short, we calculated the branching ratios of  $B_d^0 \rightarrow J/\Psi\eta$  and  $B_d^0 \rightarrow J/\Psi\eta'$  decays at the leading order by using the pQCD factorization approach. Besides the usual factorizable diagrams, the non-factorizable spectator diagrams are also calculated analytically in the pQCD approach. By keeping the transverse momentum  $k_T$ , the end-point singularity disappears in our calculation.

From our calculations and phenomenological analysis, we found the following results:

- Using the quark-flavor scheme, the pQCD predictions for the branching ratios are

$$Br(B_d^0 \rightarrow J/\Psi\eta) = (1.96_{-0.65}^{+9.68}) \times 10^{-6}, \quad (20)$$

$$Br(B_d^0 \rightarrow J/\Psi\eta') = (1.09_{-0.25}^{+3.76}) \times 10^{-6}, \quad (21)$$

where the various errors as specified previously have been added in quadrature.

- The major theoretical errors of the pQCD predictions are induced by the uncertainties of the hard energy scale  $t_i$ 's and the parameters  $\omega_b$ .

## Acknowledgments

X. Liu would like to acknowledge the financial support of The Scientific Research Start-up Fund of Zhejiang Ocean University under Grant No.21065010706. This work was partially supported by the National Natural Science Foundation of China under Grant No.10575052, and by the Specialized Research Fund for the Doctoral Program of Higher Education (SRFDP) under Grant No. 20050319008.

## APPENDIX A: RELATED FUNCTIONS

We show here the function  $h_i$ 's, coming from the Fourier transformations of the function  $H^{(0)}$ ,

$$\begin{aligned} h_e(x_1, x_3, b_1, b_3) = & K_0 \left( \sqrt{x_1 x_3 (1 - r^2)} m_B b_1 \right) \left[ \theta(b_1 - b_3) K_0 \left( \sqrt{x_3 (1 - r^2)} m_B b_1 \right) \right. \\ & \cdot I_0 \left( \sqrt{x_3 (1 - r^2)} m_B b_3 \right) + \theta(b_3 - b_1) K_0 \left( \sqrt{x_3 (1 - r^2)} m_B b_3 \right) \\ & \left. \cdot I_0 \left( \sqrt{x_3 (1 - r^2)} m_B b_1 \right) \right] S_t(x_3), \end{aligned} \quad (A1)$$

$$h_f(x_1, x_2, x_3, b_1, b_2) = \left\{ \theta(b_2 - b_1) I_0(M_B \sqrt{x_1 x_3 (1 - r^2)} b_1) K_0(M_B \sqrt{x_1 x_3 (1 - r^2)} b_2) \right. \\ \left. + (b_1 \leftrightarrow b_2) \right\} \cdot \begin{cases} K_0(M_B F_{(1)} b_2), & \text{for } F_{(1)}^2 > 0 \\ \frac{\pi i}{2} H_0^{(1)}(M_B \sqrt{|F_{(1)}^2|} b_2), & \text{for } F_{(1)}^2 < 0 \end{cases}, \quad (\text{A2})$$

where  $J_0$  is the Bessel function,  $K_0$  and  $I_0$  are the modified Bessel functions with  $K_0(-ix) = -(\pi/2)Y_0(x) + i(\pi/2)J_0(x)$ , and  $F_{(j)}$ 's are defined by

$$F_{(1)}^2 = (x_1 - x_2)x_3(1 - r^2) + r_c^2, \quad (\text{A3})$$

$$F_{(2)}^2 = (x_1 - x_2)x_3(1 - r^2) + r_c^2. \quad (\text{A4})$$

The threshold resummation form factor  $S_t(x_i)$  is adopted from Ref. [17]

$$S_t(x) = \frac{2^{1+2c}\Gamma(3/2+c)}{\sqrt{\pi}\Gamma(1+c)}[x(1-x)]^c, \quad (\text{A5})$$

where the parameter  $c = 0.3$ . This function is normalized to unity.

The Sudakov factors used in the text are defined as

$$S_{ab}(t) = s\left(x_1 m_B / \sqrt{2}, b_1\right) + s\left(x_3 m_B / \sqrt{2}, b_3\right) + s\left((1 - x_3) m_B / \sqrt{2}, b_3\right) \\ - \frac{1}{\beta_1} \left[ \ln \frac{\ln(t/\Lambda)}{-\ln(b_1 \Lambda)} + \ln \frac{\ln(t/\Lambda)}{-\ln(b_3 \Lambda)} \right], \quad (\text{A6})$$

$$S_{cd}(t) = s\left(x_1 m_B / \sqrt{2}, b_1\right) + s\left(x_2 m_B / \sqrt{2}, b_2\right) + s\left((1 - x_2) m_B / \sqrt{2}, b_2\right) \\ + s\left(x_3 m_B / \sqrt{2}, b_1\right) + s\left((1 - x_3) m_B / \sqrt{2}, b_1\right) \\ - \frac{1}{\beta_1} \left[ 2 \ln \frac{\ln(t/\Lambda)}{-\ln(b_1 \Lambda)} + \ln \frac{\ln(t/\Lambda)}{-\ln(b_2 \Lambda)} \right], \quad (\text{A7})$$

where the function  $s(q, b)$  are defined in the Appendix A of Ref. [16]. The scale  $t_i$ 's in the above equations are chosen as

$$t_e^1 = a_t \cdot \max(\sqrt{x_3(1 - r^2)} M_B, 1/b_1, 1/b_3), \\ t_e^2 = a_t \cdot \max(\sqrt{x_1(1 - r^2)} M_B, 1/b_1, 1/b_3), \\ t_f = a_t \cdot \max(\sqrt{x_1 x_3(1 - r^2)} M_B, \sqrt{(x_1 - x_2)x_3(1 - r^2) + r_c^2} M_B, 1/b_1, 1/b_2), \quad (\text{A8})$$

where  $a_t = 1.0 \pm 0.2$  and  $r = M_{J/\Psi}/M_B$ . The scale  $t_i$ 's are chosen as the maximum energy scale appearing in each diagram to kill the large logarithmic radiative corrections.

## APPENDIX B: INPUT PARAMETERS AND WAVE FUNCTIONS

The masses, decay constants, QCD scale and  $B_d^0$  meson lifetime are

$$\Lambda_{\overline{\text{MS}}}^{(f=4)} = 250 \text{ MeV}, \quad f_\pi = 130 \text{ MeV}, \quad f_{J/\Psi} = 405 \text{ MeV}, \\ m_0^{\eta_{d\bar{d}}} = 1.08 \text{ GeV}, \quad M_{B_d^0} = 5.28 \text{ MeV}, \quad M_{J/\Psi} = 3.097 \text{ GeV}, \\ M_W = 80.41 \text{ GeV}, \quad \tau_{B_d^0} = 1.54 \times 10^{-12} \text{ s}. \quad (\text{B1})$$



For the CKM matrix elements, here we adopt the Wolfenstein parametrization for the CKM matrix, and take  $\lambda = 0.2272$ ,  $A = 0.818$ ,  $\rho = 0.221$  and  $\eta = 0.340$  [4].

For the  $B$  meson wave function, we adopt the model

$$\phi_B(x, b) = N_B x^2 (1-x)^2 \exp \left[ -\frac{M_B^2 x^2}{2\omega_b^2} - \frac{1}{2}(\omega_b b)^2 \right], \quad (\text{B2})$$

where  $\omega_b$  is a free parameter and we take  $\omega_b = 0.40 \pm 0.05$  GeV in numerical calculations, and  $N_B = 91.745$  is the normalization factor for  $\omega_b = 0.40$  for the  $B$  meson.

The wave function for  $d\bar{d}$  components of  $\eta^{(\prime)}$  meson is given by

$$\Phi_{\eta_{d\bar{d}}}(p, x, \zeta) \equiv \frac{i\gamma_5}{\sqrt{2N_c}} \left[ \not{p} \phi_{\eta_{d\bar{d}}}^A(x) + m_0^{\eta_{d\bar{d}}} \phi_{\eta_{d\bar{d}}}^P(x) + \zeta m_0^{\eta_{d\bar{d}}} (\not{p} \not{\gamma} - v \cdot n) \phi_{\eta_{d\bar{d}}}^T(x) \right], \quad (\text{B3})$$

where  $p$  and  $x$  are the momentum and the momentum fraction of  $\eta_{d\bar{d}}$  respectively, while  $\phi_{\eta_{d\bar{d}}}^A$ ,  $\phi_{\eta_{d\bar{d}}}^P$  and  $\phi_{\eta_{d\bar{d}}}^T$  represent the axial vector, pseudoscalar and tensor components of the wave function respectively. We here assume that the wave function of  $\eta_{d\bar{d}}$  is same as the  $\pi$  wave function based on SU(3) flavor symmetry. The parameter  $\zeta$  is either  $+1$  or  $-1$  depending on the assignment of the momentum fraction  $x$ .

The explicit expression of chiral enhancement scale  $m_0^q = m_0^{\eta_{d\bar{d}}}$  is given by [21]

$$m_0^q \equiv \frac{m_{qq}^2}{2m_q} = \frac{1}{2m_q} [m_\eta^2 \cos^2 \phi + m_{\eta'}^2 \sin^2 \phi - \frac{\sqrt{2}f_s}{f_q} (m_{\eta'}^2 - m_\eta^2) \cos \phi \sin \phi], \quad (\text{B4})$$

and numerically  $m_0^q = 1.07 \text{ MeV}$  for  $m_\eta = 547.5 \text{ MeV}$ ,  $m_{\eta'} = 957.8 \text{ MeV}$ ,  $f_q = 1.07 f_\pi$ ,  $f_s = 1.34 f_\pi$  and  $\phi = 39.3^\circ$ .

For the distribution amplitude  $\phi_{\eta_q}^A$ ,  $\phi_{\eta_q}^P$  and  $\phi_{\eta_q}^T$ , we utilize the results for  $\pi$  meson obtained from the light-cone sum rule [23] including twist-3 contributions:

$$\begin{aligned} \phi_{\eta_q}^A(x) &= \frac{3}{\sqrt{2N_c}} f_q x(1-x) \left\{ 1 + a_2^{\eta_q} \frac{3}{2} [5(1-2x)^2 - 1] \right. \\ &\quad \left. + a_4^{\eta_q} \frac{15}{8} [21(1-2x)^4 - 14(1-2x)^2 + 1] \right\}, \end{aligned} \quad (\text{B5})$$

$$\begin{aligned} \phi_{\eta_q}^P(x) &= \frac{1}{2\sqrt{2N_c}} f_q \left\{ 1 + \frac{1}{2} \left( 30\eta_3 - \frac{5}{2}\rho_{\eta_q}^2 \right) [3(1-2x)^2 - 1] \right. \\ &\quad \left. + \frac{1}{8} \left( -3\eta_3\omega_3 - \frac{27}{20}\rho_{\eta_q}^2 - \frac{81}{10}\rho_{\eta_q(s)}^2 a_2^{\eta_q} \right) [35(1-2x)^4 - 30(1-2x)^2 + 3] \right\}, \end{aligned} \quad (\text{B6})$$

$$\begin{aligned} \phi_{\eta_q}^T(x) &= \frac{3}{\sqrt{2N_c}} f_q (1-2x) \\ &\quad \cdot \left[ \frac{1}{6} + \left( 5\eta_3 - \frac{1}{2}\eta_3\omega_3 - \frac{7}{20}\rho_{\eta_q}^2 - \frac{3}{5}\rho_{\eta_q}^2 a_2^{\eta_{q,s}} \right) (10x^2 - 10x + 1) \right], \end{aligned} \quad (\text{B7})$$

with the updated Gegenbauer moments [24]

$$a_2^{\eta_q} = 0.115, \quad a_4^{\eta_q} = -0.015, \quad \rho_{\eta_q} = 2m_q/m_{qq}, \quad \eta_3 = 0.015, \quad \omega_3 = -3.0. \quad (\text{B8})$$

---

[1] B. Aubert *et al.* (BaBar Collaboration), Phys. Rev. Lett. **98**, 131803 (2007).

- [2] A. Deandrea *et al.*, Phys. Lett. B **318**, 549 (1993).
- [3] P.Z. Skands, J. High Energy Phys. **0101** (2001) 008.
- [4] W.-M. Yao *et al.* ( Particle Data Group), J. Phys. G **33**, 1 (2006).
- [5] Heavy Flavor Averaging Group, E. Barberio *et al.*, hep-ex/0603003; and online update at <http://www.slac.stanford.edu/xorg/hfag>.
- [6] G. Buchalla, A.J. Buras, and M.E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).
- [7] G.P. Lepage and S.J. Brodsky, Phys. Rev. D **22**, 2157 (1980).
- [8] C.-H. V. Chang and H.N. Li, Phys. Rev. D **55**, 5577 (1997); T.-W. Yeh and H.N. Li, Phys. Rev. D **56**, 1615 (1997).
- [9] H.N. Li, Prog.Part.& Nucl.Phys. **51**, 85 (2003), and reference therein. H.N. Li and H.L. Yu, Phys. Rev. Lett. **74**, 4388 (1995); Phys. Lett. B **353**, 301 (1995); Phys. Rev. D **53**, 2480 (1996).
- [10] X. Liu, H.S. Wang, Z.J. Xiao, L.B. Guo and C.D. Lü, Phys. Rev. D **73**, 074002 (2006); H.S. Wang, X. Liu, Z.J. Xiao, L.B. Guo and C.D. Lü, Nucl. Phys. B **738**, 243 (2006); Z.J. Xiao, X.F. Chen and D.Q. Guo, Eur.Phys.J. C **50** (2007) in press; Z.J. Xiao, D.Q. Guo and X.F. Chen, Phys. Rev. D **75**, 014018 (2007); Z.J. Xiao, X. Liu and H.S. Wang, Phys. Rev. D **75**034017 (2007); Z.J. Xiao, X.F. Chen and D.Q. Guo, hep-ph/0701146.
- [11] A. Ali, G. Kramer, Y. Li, C.D. Lü, Y.L. Shen, W. Wang, and Y.M. Wang, hep-ph/0703162.
- [12] Y. Li and C.D. Lü, J. Phys. G **29**, 2115 (2003); High Energy & Nucl.Phys. **27**, 1061 (2003).
- [13] Y. Li, C.D. Lü, and Z.J. Xiao, J. Phys. G **31**, 273 (2005).
- [14] H.N. Li, Phys. Rev. D **66**, 094010 (2002).
- [15] H.N. Li and B. Tseng, Phys. Rev. D **57**, 443, (1998).
- [16] C.-D. Lü, K. Ukai and M.Z. Yang, Phys. Rev. D **63**, 074009 (2001).
- [17] T. Kurimoto, H.N. Li, and A.I. Sanda, Phys. Rev. D **65**, 014007 (2002); Phys. Rev. D **67**, 054028 (2003).
- [18] A.E. Bondar and V.L. Chernyak, Phys. Lett. B **612**, 215(2005).
- [19] Th. Feldmann, P. Kroll and B. Stech, Phys. Rev. D **58**, 114006 (1998).
- [20] R. Escribano and J.M. Frere, J. High Energy Phys. **0506** (2005) 029; J. Schechter, A. Subbaraman and H. Weigel, Phys. Rev. D **48**, 339 (1993)
- [21] Y.-Y. Charng, T. Kurimoto, H.N. Li, Phys. Rev. D **74**, 074024 (2006).
- [22] R. Escribano, J. Nadal, hep-ph/0703187.
- [23] P. Ball, J. High Energy Phys. **9809**, 005 (1998); P. Ball, J. High Energy Phys. **9901**, 010 (1999).
- [24] P. Ball and R. Zwicky, Phys. Rev. D **71**, 014015 (2005); P. Ball, V.M. Braun, and A. Lenz, J. High Energy Phys. **0605** (2006) 004.